Assignment Discovery Online Curriculum

Lesson title:
Numbers in Nature

Grade level:
9-12

Subject area:
Mathematics

Duration:
Two class periods

Objectives:
Students will
- understand what the Fibonacci sequence is; and
- understand how the Fibonacci sequence is expressed in nature.

Materials:
- Computers with Internet access (optional but very helpful)
- Pencils and paper
- Ruler
- Compass
- Copies of Classroom Activity Sheet: Finding Fibonacci Numbers in Nature
- Copies of Take-Home Activity Sheet: Creating the Fibonacci Spiral
- Answer Sheet for the Take-Home Activity Sheet (for teacher only)

Procedures:

1. Begin the lesson by discussing the Fibonacci sequence, which was first observed by the Italian mathematician Leonardo Fibonacci in 1202. He was investigating how fast rabbits could breed under ideal circumstances. In developing the problem, he made the following assumptions:
   - Begin with one male and one female rabbit. Rabbits can mate at the age of one month, so by the end of the second month, each female can produce another pair of rabbits.
   - The rabbits never die.
   - The female produces one male and one female every month.

Fibonacci asked how many *pairs* of rabbits would be produced in one year.

2. Work with the class to see whether students can develop the sequence themselves. Remind them that they're counting pairs of rabbits, not individual rabbits. You may want to walk them through the first few months of the problem:
   a. You begin with one pair of rabbits (1).

b. At the end of the first month, there is still only one pair (1).
c. At the end of the second month, the female has produced a second pair, so there are 2 pairs (2).
d. At the end of the third month, the original female has produced another pair, so now there are 3 pairs (3).
e. At the end of the fourth month, the original female has produced yet another pair, and the female born two months earlier has produced her first pair, making a total of 5 pairs (5).

3. Write the pattern that has emerged in step 2 on the board: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233. Discuss what "rule" is being followed to get from one number to the next. Help students understand that to get the next number in the sequence, you have to add the previous two numbers. Explain that this sequence is known as the *Fibonacci sequence*. The term that mathematicians use for the type of rule followed to obtain the numbers in the sequence is *algorithm*. As a class, continue the sequence for the next few numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987…

4. Tell students that the Fibonacci sequence has intrigued mathematicians for centuries. What's more, mathematicians have noticed that these numbers appear in many different patterns in nature, often creating the beauty we admire. Tell students that they are going to look for Fibonacci numbers in pictures of objects from nature. Make sure that students understand that they are looking for specific numbers that appear in the sequence, not for the entire sequence.

5. Divide students into groups of three or four. Distribute the Classroom Activity Sheet: Finding Fibonacci Numbers in Nature. Tell students to work together to try to answer the questions on the sheet. Make sure that each student fills out his or her own sheet. For your information, the questions and explanations are listed below. It may be helpful to work on the first example as a class so students understand what they are looking for.

   a. **Flower petals** (Provide illustrations of lilies, irises, buttercups, asters, and black-eyed Susans.) Count the number of petals on each of these flowers. What numbers do you get? Are these Fibonacci numbers? *(Lilies and irises have 3 petals, buttercups have 5 petals, and asters and black-eyed Susans have 21 petals; all are Fibonacci numbers.)*

   b. **Seed heads** (Provide an enlarged illustration of a seed head drawn from the example on the Web site http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html.) Each circle on the enlarged illustration represents a seed head. Look closely at the illustration. Do you see how the circles form spirals? Start from the center, which is marked in black. Find a spiral going toward the right. How many seed heads can you count in that spiral? Now find a spiral going toward the left. How many seed heads can you count there? Are they Fibonacci numbers? *(The numbers of* *seed heads vary, but they are all Fibonacci numbers. For example, the spirals at the far edge of the picture going in both directions contain 34 seed heads.)*

c. **Cauliflower florets** (Provide an actual cauliflower or a close-up picture of a cauliflower that shows the florets.) Locate the center of the head of cauliflower. Count the number of florets that make up a spiral going toward the right. Then count the number of florets that make up a spiral going toward the left. Are the numbers of florets that make up each spiral Fibonacci numbers? *(The numbers of florets will vary, but they should all be Fibonacci numbers.)*

d. **Pinecone** (You may wish to use the enlarged picture from the Web site http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html as an example. However, you may find it easier to use an actual pinecone.) Look carefully at the picture of a pinecone. Do you see how the seed cases make spiral shapes? Find as many spirals as possible going in each direction. How many seed cases make up each spiral? Are they all Fibonacci numbers? *(The numbers will vary, but they should all be Fibonacci numbers.)*

e. **Apple** (Use a picture of an apple or an actual apple cut horizontally through the middle, not lengthwise, so that it shows the seeds surrounded by a star shape.) How many points do you see on the "star"? Is this a Fibonacci number? *(There are five points on the star.)*

6. Ask the class which shape emerges the most often from clusters of seeds *(the spiral).* Discuss whether there are any advantages to this shape. Explain that seeds may form spirals because this is an efficient way of packing the maximum number of seeds into a small area.

7. Ask students where else they see the spiral shape in nature (nautilus shell). Would they guess that those spirals are also formed from Fibonacci numbers? Do they find this shape pleasing to the eye? To conclude, discuss other pleasing shapes and patterns in nature, such as those of waves, leaves, and tornadoes. Discuss whether these, too, may have a mathematical basis.

8. Assign the Take-Home Activity Sheet: Creating the Fibonacci Spiral. Have students share their drawings in class. Discuss how rectangles with Fibonacci dimensions are used in art and architecture. You could use the examples of painter Piet Mondrian, who used three- and five-unit squares in his art; the Egyptians, who used Fibonacci dimensions in the Gaza Great Pyramid; and the Greeks, who used these dimensions in the Parthenon. Then brainstorm some animals with spiral features. (The shape is similar to the spiral of the snail, the nautilus, and other seashells). What function does the spiral serve? (Some scientists think it protects the animal inside the shell.)

Adaptation for younger students:
Work on the Classroom Activity Sheet together in class. Help students find the Fibonacci numbers in each illustration. Students may also enjoy working on the Take-Home Activity Sheet as a whole-class project.

Questions:

1. Imagine that scientists in the rain forest have discovered a new species of plant life. Where might they look for the Fibonacci sequence?

2. Suppose that you're shooting baskets with a friend. After a few practice shots, you decide that you want to keep score. The first basket either of you makes is worth one point. Just to make things interesting, you suggest that every time either of you makes another basket, you add your previous two scores to get a new total. To make the game even more appealing, you offer to start from zero, while your friend can start from one. What sequence of numbers would emerge after shooting eight baskets? What is the difference in points between you and your friend? What pattern has emerged from the point difference?

3. The Fibonacci sequence continues indefinitely. If all its terms were added together, it would be called a "series," and the result would be infinite. But not all infinite series add up to infinity. For example, adding all the terms of 1/n (where n is 1, 2, 3…) does not result in a large sum at all, even though the series could go on forever. What would the sum of five numbers in the series 1/n be? Explain why it wouldn't be infinitely large.

4. Explain that numbers missing from the Fibonacci sequence can be obtained by combining numbers in the sequence, assuming that you're allowed to use each number more than once. For example, how could the number 4 be obtained from the sequence? How about 11? 56? Think of a number not in the sequence and try to figure out what numbers to combine to get it.

5. At first glance, the natural world may appear to be a random mixture of shapes and numbers. On closer inspection, however, we can spot repeating patterns like the Fibonacci numbers. Are humans more apt to perceive some patterns than others? What makes certain patterns more appealing than others?

6. Try to solve this problem: Female honeybees have two parents, a male and a female, but male honeybees have just one parent, a female. Can you draw a family tree for a male and a female honeybee? What pattern emerges? Are they Fibonacci numbers?

   (The male bee has 1 parent, and the female bee has 2 parents. The male bee has 2 grandparents, and the female bee has 3 grandparents. The male bee has 3 great-grandparents, and the female bee has 5 great-grandparents. The male bee has 5 great-great-grandparents, and the female bee has 8 great-great-grandparents. The male bee has 8 great-great-great-grandparents, and the female bee has 13 great-great-great-grandparents.)

Evaluation:

You can evaluate students using the following three-point rubric:

Three points: active participation in classroom discussions; ability to work cooperatively to complete the Classroom Activity Sheet; ability to solve all the problems on the sheet

Two points: some degree of participation in classroom discussions; ability to work somewhat cooperatively to complete the Classroom Activity Sheet; ability to solve three out of five problems on the sheet

One point: small amount of participation in classroom discussions; attempt to work cooperatively to complete the Classroom Activity Sheet; ability to solve one problem on the sheet

Extensions:

Finding Ratios

Suggest that students measure the length and width of the following rectangles:

a. a 3” _ 5” index card
b. an 8.5” _ 11” piece of paper
c. a 2” _ 3” school photo
d. a familiar rectangle of their choice

Have students find the ratio of length to width for each of the rectangles. Then have them take the average of all the ratios. What number do they get? (1.61803). Tell students that this ratio is called the golden ratio and that it occurs in many pleasing shapes, such as pentagons, crosses, and isosceles triangles, and is often used in art and architecture.

An Algebraic Rule

Encourage students to try to develop an algebraic formula that expresses the Fibonacci sequence. The formula is described below.

Represent the first and second terms in the sequence with x and y. Then the first few terms would be expressed as follows:

First term = x
Second term = y
Third term = (x+y)
Fourth term = (x+y)+y = **1**x + **2**y
Fifth term = (x+2y) + (x+y) = **2**x + **3**y
Sixth term = (2x+3y) + (x+2y) = **3**x + **5**y
Seventh term = 3x+5y + 2x+3y = **5**x + **8**y

Ask the students whether they notice anything familiar about the coefficients. (They're numbers in the Fibonacci sequence.)

Suggested Reading:

Life By the Numbers
Keith Devlin. John Wiley & Sons, 1998.
Written as a companion volume to the PBS series of the same name, this book focuses on the role mathematics plays in everyday life. Each chapter examines a different aspect of the world we live in and how mathematics is involved: patterns appearing in nature, the curve of a baseball, the chance of winning in Las Vegas, the technology of the future. Lots of pictures round out this clear and exciting presentation.

Designing Tessellations: The Secrets of Interlocking Patterns
Jinny Beyer. Contemporary Books, 1999.
For generations, people have created designs using repeating, interlocking patterns—tessellations. In this slightly oversized, beautifully illustrated book, the author shows how the combination of pattern and symmetry can result in stunning geometric designs. While this unique book uses quilt making as the focus of the design process, it could easily be applied to other arts as well.

Vocabulary:

algorithm
Definition: A step-by-step procedure for solving a problem.
Context: The **algorithm** for obtaining the numbers in the Fibonacci sequence is to add the previous two terms together to get the next term in the sequence.

logarithmic spiral
Definition**:** A shape that winds around a center and recedes from the center point with exponential growth.
Context: The nautilus shell is an example of a **logarithmic spiral**.

sequence
Definition: A set of elements ordered in a certain way.
Context: The terms of the Fibonacci **sequence** become progressively larger.

term
Definition: An element in a series or sequence.
Context: The mathematician Jacques Binet discovered that he could obtain each of the **terms** in the Fibonacci sequence by inserting consecutive numbers into a formula.

Academic standards:
Grade level:
9-12
Subject area:
Mathematics
Standard:
Understands and applies basic and advanced properties of the concepts of numbers.
Benchmark:
Uses discrete structures (e.g., finite graphs, matrices, or sequences) to represent and to solve problems.

Grade level:
9-12
Subject area:
Mathematics
Standard:
Uses basic and advanced procedures while performing the processes of computation.
Benchmark:
Uses recurrence relations (i.e., formulas that express each term as a function of one or more of the previous terms, such as the Fibonacci sequence and the compound interest equation) to model and to solve real-world problems (e.g., home mortgages or annuities).

Grade level:
9-12
Subject area:
Mathematics
Standard:
Uses basic and advanced procedures while performing the processes of computation.
Benchmark:
Uses a variety of operations (e.g., finding a reciprocal, raising to a power, taking a root, and taking a logarithm) on expressions containing real numbers.

Credit:
Chuck Crabtree, freelance curriculum writer.

Name: ____________________

Finding Fibonacci Numbers in Nature

Below are examples from nature in which Fibonacci numbers can be found. Using the illustrations or samples your teacher provides, work with your group to answer the questions. Make sure that you complete your own sheet.

1. Flower petals: Count the number of petals on each of these flowers. What numbers do you get? Are these Fibonacci numbers?

2. Seed heads: Each circle on the enlarged illustration represents a seed head. Look closely at the illustration. Do you see how the circles form spirals? Start from the center, which is marked in black. Find a spiral going toward the right. How many seed heads can you count in that spiral? Now find a spiral going toward the left. How many seed heads can you count there? Are they Fibonacci numbers?

3. Cauliflower florets: Locate the center of the head of cauliflower. Count the number of florets that make up a spiral going toward the right. Then count the number of florets that make up a spiral going toward the left. Are the numbers of florets that make up each spiral Fibonacci numbers?

4. Pinecone: Look carefully at the picture of a pinecone. Do you see how the seed cases make spiral shapes? Find as many spirals as you can going in each direction. How many seed cases make up each spiral? Are they all Fibonacci numbers?

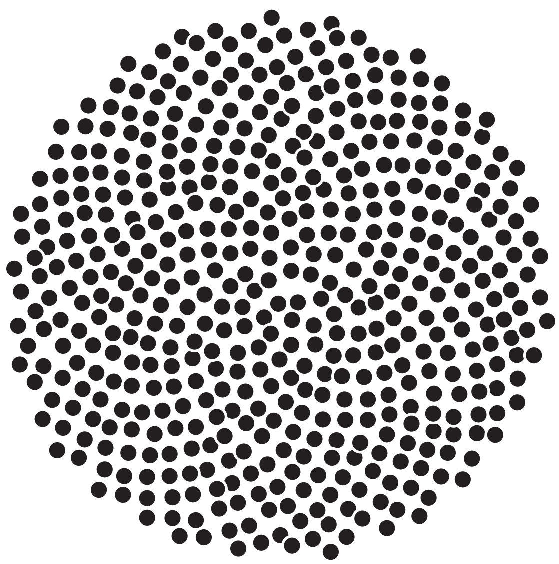
5. Apple: How many points do you see on the “star”? Is this a Fibonacci number?

What shape emerges most often from the Fibonacci numbers? What function do you think this shape serves?

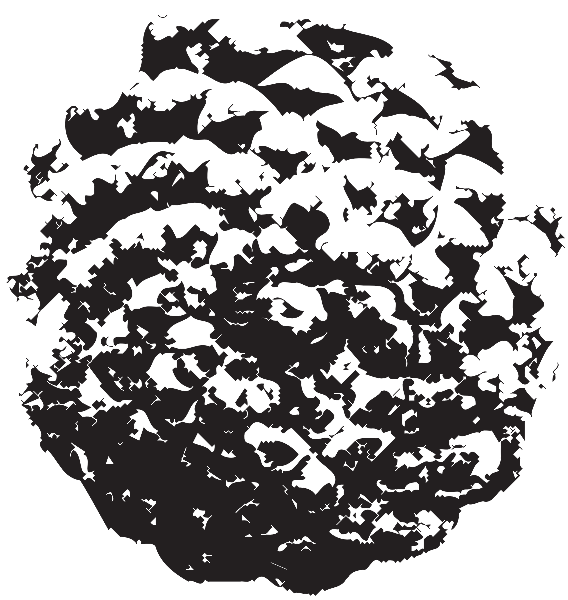
Name: ____________________

Finding Fibonacci Numbers in Nature

Seed head



Pinecone



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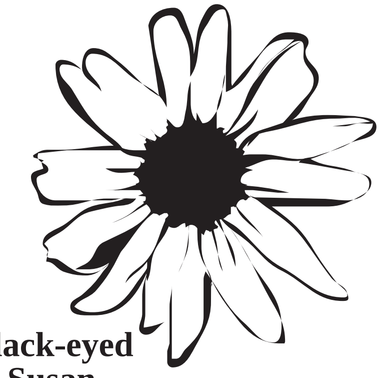
Finding Fibonacci Numbers in Nature

Buttercup

Lily

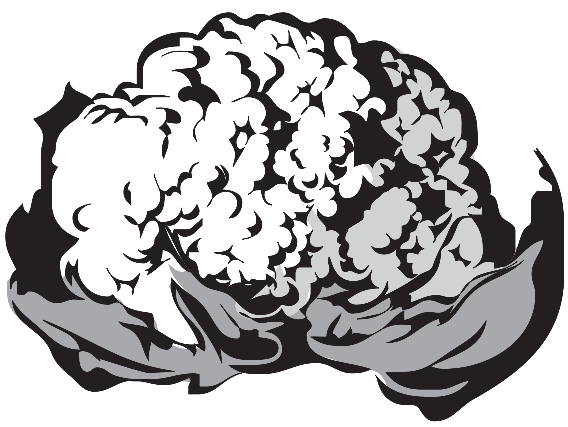


Black-eyed Susan

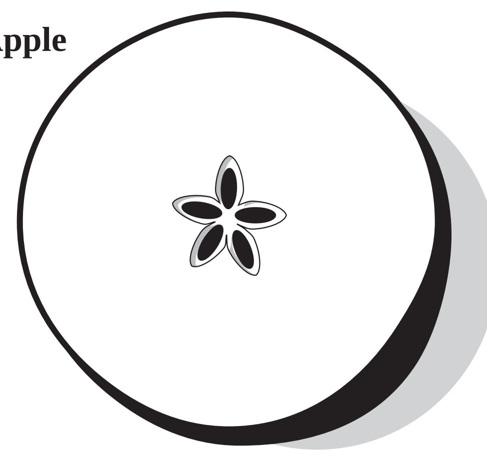
Iris

Aster

Cauliflower florets



Apple



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Creating the Fibonacci Spiral

You will need a large piece of draft paper, a ruler, and a compass to create the Fibonacci spiral. Follow the directions below and watch the spiral emerge.

1. Draw a small square that is 1 inch on each side. Draw a second square of the same size directly to the left of the first square so that the sides of the two squares are touching.

2. Draw a 2-inch square just above the two 1-inch squares.

3. Draw a 3-inch square to the right of the three smaller squares so that it borders the two 1-inch squares and the 2-inch square. All the squares should be connected.

4. Draw a 5-inch square that borders the 1-, 2-, and 3-inch squares (below the smaller squares). Each successive square will have an edge whose length is the sum of those of the two squares immediately preceding it.

5. Draw an 8-inch square to the left of the previous five squares.

6. Draw a 13-inch square above the previous six squares.

7. You will need your compass to complete your drawing. Within each square of your drawing, you are going to draw an arc, or a quarter circle, from one corner to the opposite corner. (You will be drawing an arc of a circle with a radius equal to the length of one side of that square.) Each arc will be connected to the next.

To begin, place your pencil in the upper-right-hand corner of the FIRST 1-inch square you drew and draw an arc downward to its lower-left-hand corner. Next, draw an arc from that same point on the SECOND 1-inch square (the lower-right-hand corner) to the upper-left-hand corner of the second square. Continue drawing arcs in each square you created, starting each arc at the point where the last one ended.

8. What shape did you get? What forms in nature reflect this shape?

Creating the Fibonacci Spiral

ANSWER SHEET (for the teacher only)

The drawings your students make should look like this: